Who Acquires Information in Dealer Markets?*

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Abstract

We study information acquisition in dealer markets. We first identify a one-sided strategic complementarity in information acquisition: the more informed traders are, the larger market makers' gain from becoming informed. We then fully characterize the unique equilibrium as a function of the (uniform) cost of information, and the composition of the market in terms of liquidity traders and speculators. Lastly, we examine the implications of our analysis for market liquidity and price discovery. Our findings shed light on several empirical regularities.

JEL classification: D80; D82; G10; G14; G20

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1 Introduction

The canonical model of dealer markets (Glosten and Milgrom, 1985) assumes that traders are superiorly informed. Yet empirical evidence shows that market makers (MMs) can be better informed than traders.[1] What does the structure of dealer markets imply about when we should expect to see one situation or the other?

Dealer markets are two-sided financial markets with MMs quoting prices on one side of the market and traders submitting market orders on the other side (see Figure 1); traders benefit from MMs competing to offer the best price, while MMs make up with "liquidity traders" the losses incurred from trading against "speculators". The structure of dealer markets determines the respective trading opportunities of traders and MMs, and, indirectly, incentives to acquire information on each side of the market. Suppose that the cost of acquiring information is the same for all market participants, who then becomes informed? To address this question, we investigate a two-stage model where information acquisition takes place in the first period, and trade in the second.

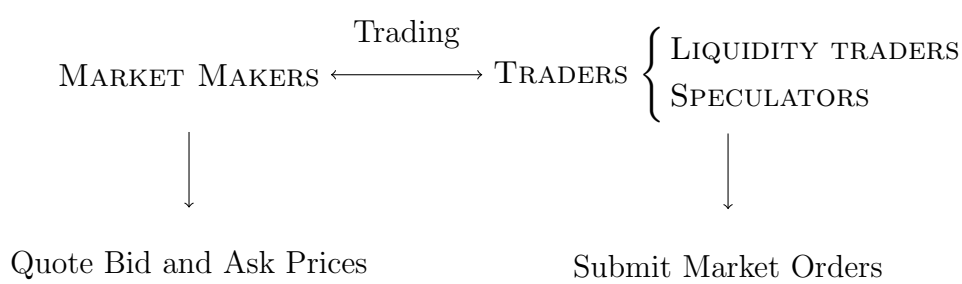


Figure 1: Dealer Markets

We first identify a one-sided strategic complementarity in information acquisition: *the MMs' gain from becoming informed is increasing in the probability that traders are informed.* By contrast, traders' gain from becoming informed is always decreasing in the probability that MMs are informed. The underlying logic of this one-sided complementarity is as follows. The greater the number of informed traders the worse the adverse selection problem of uninformed MMs. This leads the latter to set larger spreads. Informed MMs on the other hand are protected from adverse selection, but benefit from the larger spreads of uninformed MMs

[1] See the discussion of the empirical evidence later in this introduction.

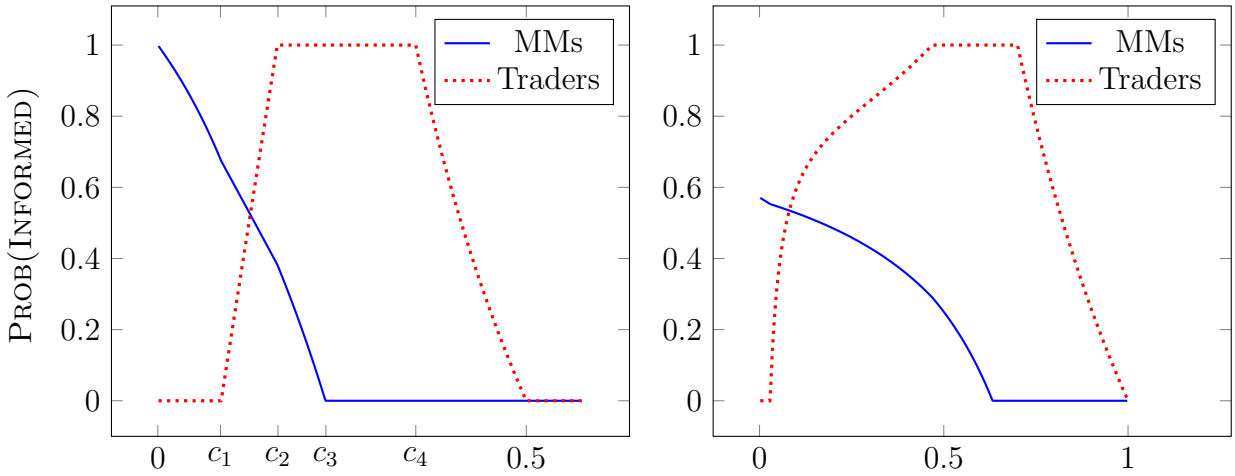
since this allows them to increase their own spreads. Consequently, the difference between profits made by informed MMs and uninformed MMs increases with the probability of informed trading.

We then draw the implications with respect to equilibrium information acquisition when the cost of information is the same for traders and MMs. In equilibrium, the profile of information acquisition is uniquely determined by the parameters of the model, that is: (a) the cost of information and (b) the composition of the market in terms of liquidity traders and speculators.[2] We fully characterize the equilibrium as a function of the parameters.

Who acquires information varies with the cost as illustrated in Figure 2, panel A. When the cost is small, MMs acquire information whereas traders choose to remain uninformed. The reverse holds when the cost is high: traders acquire information whereas MMs choose to remain uninformed. This region of the cost therefore microfounds the standard model of Glosten and Milgrom. In an intermediate range of the cost, MMs and traders randomize between acquiring information and remaining uninformed. The logic is the following. Due to the positive fraction of liquidity traders in the market, a MM's gain from becoming informed is bounded away from zero as long as not all her competitors acquire information. Therefore the probability that MMs acquire information must approach one as the cost tends to zero. By contrast, since speculators' dealings are with MMs and never with liquidity traders, traders' gain from becoming informed is proportional to the probability that MMs are uninformed. Consequently, traders remain uninformed when the cost is low (but most MMs are informed), whereas traders acquire information when the cost is high (but most MMs are uninformed).

The effect of market composition on information acquisition is illustrated in Figure 2, panel B. When most traders are speculators (i.e. liquidity shocks are rare), traders acquire information whereas MMs remain uninformed; by contrast, when most traders are liquidity traders, MMs acquire information and traders remain uninformed. Intuitively, a speculator is either informed or abstains from trading, which means that MMs only recoup the cost of acquiring information by interacting with liquidity traders. The probability of informed market making is therefore increasing in the fraction of liquidity traders comprising the market. Information acquired by MMs in turn pins down traders' incentives to become informed (through the logic developed in the previous paragraph): traders remain uninformed when the fraction

[2] In the model, we allow traders to be hit by liquidity shocks before and/or after deciding whether or not to acquire information. Throughout the paper, we therefore refer to "traders' information acquisition" rather than "speculators' information acquisition". Traders hit by a liquidity shock before making their decision do not acquire information. However, at the time trading takes place, some liquidity traders might have already (wastefully) acquired information.



(A) Cost of Information (B) Fraction of Speculators

Figure 2: Equilibrium Information Acquisition

of liquidity traders is large (in which case most MMs are informed), whereas traders acquire information when liquidity traders are rare (in which case most MMs are uninformed).

In the final section, we examine "market liquidity" (the difference between ask and bid prices offered) and "price discovery" (how well prices reflect assets' true values). Market liquidity is non-monotonic in the cost of information. This is due to the fact that MMs become informed with probability approaching one as the cost approaches zero, whereas no market participant acquires information when the cost is very large (see Figure 2). The spreads therefore vanish at both extremes of the cost. In between, however, a combination of informed trading *and* informed market making induces positive spreads.

We then show two surprising phenomena relating to price discovery: *the extent to which prices reflect asset values can increase with (a) the cost of information and (b) the fraction of liquidity traders comprising the market.* (a) is a consequence of the fact that, in an intermediate range of information cost, raising the cost pushes MMs to acquire less information but induces traders to acquire more information. Due to the strategic complementarity in information acquisition previously highlighted, the second effect is the dominant force: the net impact of an increase in the cost is to improve price discovery. (b) is due to the fact that, when the cost is low, increasing the mass of liquidity traders enhances information acquisition from MMs. Competition between informed MMs then pushes prices toward the asset's true value.

Our findings shed light on several well-documented empirical regularities:

- First, *both traders and MMs may have proprietary information.* Manaster and Mann (1996), for instance, provide evidence in connection with the market for commodity futures, Li and Heidle (2004) for stockmarkets, and Covrig and Melvin (2002) and Sapp (2002) for the foreign exchange market. Therefore, traders cannot be viewed purely as uninformed liquidity traders, and MMs cannot be viewed as only learning from their private knowledge of the order flow.[3]

- Second, *dealer-driven price discovery can be more important than trader-driven price discovery.* For stock markets, Anand and Subrahmanyam (2008) find that "intermediaries appear to be more informed than all other institutions and individuals combined". Valseth (2013) explores government bond markets and compares the informational content of the interdealer and customer order flows: the interdealer order flow explains almost a quarter of daily yield variation, whereas the customer order flow has little explanatory power.

- Third, *MMs are often asymmetrically informed.* This has been widely documented (Albanesi and Rindi, 2000; Huang, 2002; Massa and Simonov, 2003). In our setting, ex ante identical MMs play mixed information acquisition strategies in equilibrium, and may therefore be ex-post asymmetrically informed.

- Fourth, *more volatile assets exhibit larger spreads.* Stoll (1978) was first to provide evidence in the case of stocks, while Chen, Lesmond and Wei (2007) find that spreads are higher for corporate bonds with lower rating or higher maturity, which are both associated with higher price volatility.[4] We establish that the bid-ask spread is largest when there is both informed market making and informed trading. Consequently, in our setting, spreads are maximized when prices are volatile.[5]

[3]MMs acquiring information through this channel have been considered in, for instance, Leach and Madhavan (1993), Bloomfield and O'Hara (2000) and de Frutos and Manzano (2005). In this literature, the focus is on MMs' incentives to experiment with prices in order to learn new information.

[4]Edwards, Harris and Piwowar (2007) and Bao, Pan and Wang (2011) find similar evidence, but their measure of market liquidity is different.

[5]When MMs randomize between acquiring information and not, each MM is uncertain about the information of her competitors. The price-setting equilibrium thus involves mixing on the part of all MMs.

The rest of the paper is organized as follows. Below we discuss the related literature. The model is presented in Section 2. In Section 3, we take the profile of information as given, and derive the profits made by the different market participants. Section 4 characterizes information acquired in equilibrium. In Section 5 we analyze market liquidity and price discovery. Section 6 concludes. All proofs are relegated to the appendix.

Related literature. The literature on information acquisition in financial markets stretches back at least to Grossman and Stiglitz (1980) and Verrecchia (1982). The canonical model of dealer markets is due to Glosten and Milgrom (1985), who *assume* that traders are better informed than market makers. Within this setting, endogenizing traders' information acquisition is a straightforward exercise, if one maintains the assumption that market makers are uninformed (see, e.g., Foucault, Pagano and Röell (2013)). By contrast, the problem of information acquisition by market makers is non-trivial. If one fixes traders' information, the problem is formally equivalent to information acquisition in a standard (first-price sealed-bid common-value) auction setting, analyzed in e.g. Milgrom (1981), Lee (1984), Persico (2000), and more recently in Atakan and Ekmekci (2017). To the best of our knowledge, the present paper is the first to analyze information acquisition occuring on both sides of a dealer market, and to examine how information acquired by one side of the market affects incentives to acquire information on the other.

Within the literature on dealer markets, Chamley (2007) allows traders to acquire costly information.[6] Leach and Madhavan (1993), Bloomfield and O'Hara (2000), and de Frutos and Manzano (2005) on the other hand take traders' information as given, and explore market makers' incentives to manipulate prices in order to learn from the order flow. Our work also relates to Biais (1993), who considers market makers with private information about inventories, and to Calcagno and Lovo (2006), who explore price competition when one market maker possesses superior information about fundamentals. Moinas (2010) and Boulatov and George (2013) analyze the choice of informed traders between supplying and demanding liquidity. However, all of these papers take the information of market makers as exogenous.

[6]Glode, Green and Lowery (2012) and Bolton, Santos and Scheinkman (2016) also examine information acquisition but in very different contexts.

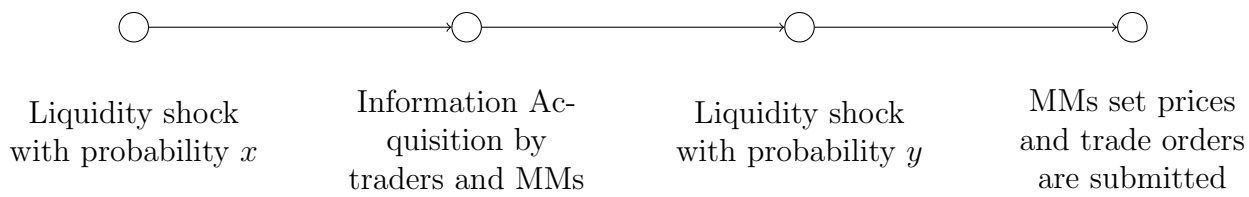


FIGURE 3: TIMING

2 Model

We consider the market for a risky asset with value V. For simplicity, $V = 1$ with probability $\frac{1}{2}$ and $V = 0$ with probability $\frac{1}{2}$. We focus throughout the paper on two market makers (MMs, indexed by $n = 1, 2$) and one trader all involved in a single trading round. In practice, of course, assets are traded multiple times by each market participant. Focusing on a single trading round enables us to pin down the exact gains from becoming informed and to derive concise results.

There are two periods. At $t = 1$, all market participants decide whether to privately learn the realization of V, for a cost $c > 0$. At $t = 2$, trade takes place. The trader decides whether to submit a market order for one unit of the asset, and MMs simultaneously set ask and bid prices a_n and b_n. In particular, neither the trader nor the MMs observe quotes before trading. There is price priority on the market; whenever MMs are tied, we determine the trading probabilities as part of the equilibrium.[7] Thus, a trader's profit from a buy order (resp. a sell order) is $V - \hat{a}$ (resp. $\hat{b} - V$), where $\hat{a} := \min_n a_n$ and $\hat{b} := \max_n b_n$; the profit of the MM who carries out the order is opposite.

The trader is hit by a liquidity shock with probability x before her decision in period $t = 1$, and with (independent) probability y after this decision. Whether or not the trader is hit is private information. A trader who is not hit by any of the liquidity shocks becomes a *speculator*. A speculator trades so as to maximize expected profit. The (ex ante) mass of speculators is denoted by π, i.e.

$$\pi := (1 - x)(1 - y).$$

A trader who suffers a liquidity shock becomes a *liquidity trader*. By definition, a liquidity

[7] We can think of this as brokers having preferred dealers to which they submit orders as long as this dealer has the best (but not necessarily strictly best) price on the market. Thus, the probability that n wins the order if there is a tie, can be thought of as the probability that the broker submits the order to the dealer.

trader buys and sells the asset with probability $\frac{1}{2}$ independently of the value V. To make the analysis interesting we assume that $\pi \in (0, 1)$. Figure 3 summarizes the timing.

Let p_n denote the probability that MMn acquires information at $t = 1$. MMn's strategy specifies p_n, and pricing strategies conditional on her information. The trader's strategy specifies a probability q of acquiring information at $t = 1$ in case no liquidity shock occurred before $t = 1$, as well as the trader's demand for the asset in period $t = 2$ in case the trader becomes a speculator. The equilibrium concept is perfect Bayesian equilibrium. Since the bid and ask sides of the market are symmetric we focus on the bid side throughout the paper.

Discussion of Assumptions. A number of simplifying assumptions can be relaxed without affecting the substance of our analysis. This includes assumptions concerning the number of MMs, on the binary nature of the asset value, and on the symmetry between the bid and ask sides of the market.

By contrast, the assumption that all quotes and trade orders are submitted simultaneously at $t = 2$ is substantial, and delimits the subset of markets to which our analysis applies. This assumption has two interpretations: either the market is fragmented, as in e.g. Biais (1993), and traders do not observe quotes but submit orders to a broker who executes at the best price; or the quotes constitute hidden liquidity, as in e.g. Boulatov and George (2013). If instead a MM could observe her competitor's quotes she could make inferences about the latter's information, and use that information to improve her own bid. Free-riding would then reduce MMs' incentives to acquire information. Similarly, if a speculator could observe quotes before trading she could then infer something from the quotes of informed MMs and exploit that information by trading with uninformed MMs.

3 A One-Sided Strategic Complementarity

We fix in this section the profile of information acquisition, (p_1, p_2, q), and analyse the *price-setting equilibrium* in period $t = 2$ when each market participant makes correct inferences concerning the probabilities with which others become informed. We identify a one-sided strategic complementarity in information acquisition: the greater the probability that traders are informed the larger MMs' gain from becoming informed. To shorten notation let henceforth MMnU denote the uninformed type of MMn, MMnH the informed type who has observed $V = 1$, and MMnL the informed type who has observed $V = 0$.

For $q = 0$, MMs' price setting is equivalent to a first-price sealed-bid common-value auction with asymmetrically informed bidders. Auctions of this kind have been extensively studied in the literature, starting with Engelbrecht-Wiggans, Milgrom and Weber (1983).[8] In Lemma 1 of the appendix we characterize the (unique) price-setting equilibrium for all (p_1, p_2, q). In this equilibrium, an uninformed speculator abstains from trading because $\hat{b} \leq \frac{1}{2} \leq \hat{a}$. An informed speculator observing $V = 1$ buys, and an informed speculator observing $V = 0$ sells. When MMs are neither both informed nor both uninformed, the price-setting equilibrium is in mixed strategies. The support of the bidding strategy from different types of a given MM never strictly overlap: MMnL bids below MMnU, who himself bids below MMnH. Details of the analysis are relegated to the appendix. The novelty of our model is that information acquisition occurs on both sides of the market, that is both from traders and MMs. In this paper, the focus of our attention is therefore on the relative size of the profits made by different market participants, and the effect of information acquired by one side of the market on incentives to acquire information on the other side.

We first compare the gains from acquiring information for MM1 and MM2. Let $\Pi_n(p_1, p_2, q)$ denote the expected profit of MMn in the price-setting equilibrium when MMn is uninformed. Similarly, let $\overline{\Pi}_n(p_1, p_2, q)$ denote the expected *gross* profit of MMn when informed, i.e. the expected profit before subtracting the cost of information c. Henceforth, we shall use profit to refer to gross profit. These profits are calculated before knowing the trade order. Hence, $\overline{\Pi}_n - \Pi_n$ represents MMn's expected gain from becoming informed, given the profile of information acquisition.

Proposition 1. *Fix* (p_1, p_2, q). *If* $p_n > p_m$ *then* $\Pi_m = 0 < \Pi_n < \overline{\Pi}_n = \overline{\Pi}_m$.

Suppose $p_2 > p_1$. By the proposition, $\Pi_1 = 0 < \Pi_2 < \overline{\Pi}_2 = \overline{\Pi}_1$. First, no matter if MM2 acquires information with much higher probability than MM1, both earn the same profit when informed. Since both know the asset value, neither MM1H nor MM2H face any adverse selection. Thus, if MM2H were to make a higher profit than MM1H, MM1H could just bid marginally higher than MM2H's highest bid, thereby winning an incoming order with probability one and obtaining the same profit as MM2H. Second, MM2U makes strictly greater expected profit than MM1U. The reason is that MM1U faces more adverse selection than MM2U, since there is a higher probability that he is competing with an informed MM. Proposition 1 thus shows that *an uninformed MM extracts rent from competitors' belief that*

[8] See also Lee (1984); Hausch and Li (1993); Calcagno and Lovo (2006); Syrgkanis, Kempe and Tardos (2013).

he is informed with higher probability.

We next record the implication of Proposition 1 for information acquisition: it cannot be that MMs acquire information with different probabilities. Indeed, if $p_n > p_m$, then $\overline{\Pi}_n - \Pi_n < \overline{\Pi}_m - \Pi_m \leq c$, which would contradict equilibrium behavior.

Corollary 1. *In equilibrium, all MMs acquire information with the same probability.*

In view of Corollary 1 we henceforth focus on profiles of information acquisition satisfying $p_1 = p_2 =: p$. Extending previous notation, let $\Pi_n(p, q)$ denote the expected profit of MMn when uninformed, and $\overline{\Pi}_n(p, q)$ the expected profit of MMn when informed. Clearly, a trader hit by a liquidity shock before $t = 1$ never acquires information. The focus of our analysis is therefore on traders who at $t = 1$ have not yet been hit by a liquidity shock. Let $\Pi_S(p, q)$ denote the expected profit of such a trader when uninformed, and $\overline{\Pi}_S(p, q)$ denote the expected profit when informed. The following theorem summarizes the effect of the pattern of information acquisition on each agent's gain from acquiring information.

Theorem 1.

1. $\overline{\Pi}_n(p, q) - \Pi_n(p, q)$ *is continuous, decreasing in* p*, and increasing in* q*;*

2. $\overline{\Pi}_S(p, q) - \Pi_S(p, q)$ *is continuous, decreasing in* p *and* q*.*

Information acquired by MMs enhances competition for profitable market orders, and ultimately reduces the profits made by informed MMs. Increasing p also reduces a speculator's chance of facing an uninformed MM. Hence, the higher p the smaller the gains from becoming informed, for *all* market participants. The effects of increasing q are more subtle. First, increasing q worsens the adverse selection problem of uninformed MMs. It therefore induces uninformed MMs to set larger spreads, which unambiguously lowers the profits of informed speculators. However, this now allows informed MMs to increase their own spreads. Consequently, increasing q raises the profits made by informed MMs, inducing a one-sided strategic complementarity in information acquisition: MMs' gain from becoming informed is increasing in the probability that traders are informed.

4 Who Acquires Information

In this section, we examine equilibrium patterns of information acquisition. We show that for any set of parameters (x, y, and c), the pattern of information acquisition is uniquely

determined. However, who acquires information varies as a function of the parameters. The next theorem characterizes the equilibrium and forms the basis of our analysis.

Theorem 2. *There exists a unique equilibrium, with the following characteristics. The probability that MMs acquire information is non-increasing in the cost of information, and tends to* 1 *as the cost of information tends to* 0*. If* $y \geq \frac{2-x}{3-x}$ *traders never acquire information* ($q = 0$)*, no matter the cost. If* $y < \frac{2-x}{3-x}$*, there exist* $0 < \underline{c} < \overline{c} \leq \frac{1-y}{2}$ *such that:*

- *for* $c \in (0, \underline{c})$ *MMs acquire information but traders remain uninformed* ($p > 0$ *and* $q = 0$)*.*
- *for* $c \in (\underline{c}, \overline{c})$*, both MMs and traders acquire information* ($p > 0$ *and* $q > 0$)*.*
- *for* $c \in (\overline{c}, \frac{1-y}{2})$ *traders acquire information but MMs remain uninformed* ($p = 0$ *and* $q > 0$)*.*
- *for* $c \geq \frac{1-y}{2}$*, traders and MMs remain uninformed* ($p = 0$ *and* $q = 0$)*.*

The arguments proving equilibrium uniqueness, as well as the monotonicity of MMs' information acquisition as a function of c, are based on the properties shown in Theorem 1. The next step consists in showing that in the limit as c tends to zero, MMs must be acquiring information with probability close to 1. If this were not the case, then prices would not accurately reflect the asset value. This remark, together with the fact that MMs can make profits from liquidity traders, implies that MMs' gain from becoming informed is bounded away from zero as long as not all MMs become informed. Hence, as c tends to zero, MMs' probability of acquiring information must be tending to 1. Traders, however, only recoup the cost of information from trading with uninformed MMs. Thus, traders' opportunities of making a profit vanish as c tends to zero. In fact, we show that traders altogether stop acquiring information as c tends to zero. The reason is that traders' gain from becoming informed is second order in c: the probability of an uninformed MM is of order c, and so is the profit made from trading with an uninformed MM. The latter observation follows from noting that a MM who is uninformed must be bidding somewhere in the vicinity of the low value of the asset, since the residual demand he faces contains (almost) only unprofitable trades.

Lastly, the threshold $y = \frac{2-x}{3-x}$ above which traders never acquire information is calculated by comparing traders' maximum gain from becoming informed, obtained when $p = 0$ and $q = 0$ (by virtue of Theorem 1), with MMs' gain given the same profile of information acquisition. When $p = 0$ and $q = 0$, then $\hat{a} = \hat{b} = \frac{1}{2}$. Therefore, traders' gain from becoming informed is $(1-y)\frac{1}{2}$: the probability that no liquidity shock occurs is $1-y$, in which case traders can sell

(resp. buy) an asset worth 0 (resp. 1) at the price $\frac{1}{2}$. On the other hand, MMs' gain from becoming informed is $\frac{1}{4}(1-\pi)$: the probability of a liquidity trader is $1-\pi$, the probability that a liquidity trader either buys an asset worth 0 or sells an asset worth 1 is $\frac{1}{2}$, and in each case the profit made by the informed MM undercutting her competitors is $\frac{1}{2}$. Equating $(1-y)\frac{1}{2}$ and $\frac{1}{4}(1-\pi)$ yields $y = \frac{2-x}{3-x}$. Intuitively, traders' gain from becoming informed is proportional to the probability that MMs are uninformed. As c increases, MMs acquire less and less information; when y is not too high this in turn provides incentives for traders to become informed. In fact, we show in the next proposition that if y is sufficiently low then a range of c exists such that traders acquire information with probability 1, and all MMs remain uninformed. This range of the cost of information microfounds Glosten and Milgrom's original assumption.

Proposition 2. [Glosten-Milgrom Markets] *If $y < \frac{x}{1+x}$ then, in a non-empty range of the cost, traders acquire information with probability 1 whereas all MMs remain uninformed ($p = 0$ and $q = 1$).*

Figure 2, panel A, illustrates Theorem 2 and Proposition 2 for $x = 0.7$ and $y = 0$ (hence, $\pi = 0.3$).[9] The vertical axis shows information acquisition; the information cost is on the horizontal axis. The solid curve indicates the equilibrium p, and the dashed curve the equilibrium q.

We next explore the effect of the share of speculators (π) on information acquisition. Recall that $\pi := (1-x)(1-y)$. We state the next proposition by fixing y and varying π through x.[10]

Proposition 3. *Fix y. Then the probability that MMs acquire information is non-increasing in the fraction of speculators, π. If $c < \frac{1-y}{2}$, there exist $0 \leq \underline{\pi} \leq \overline{\pi} < 1$ such that:*

- *for $\pi \in (0, \underline{\pi})$ MMs acquire information but traders remain uninformed ($p > 0$ and $q = 0$).*
- *for $\pi \in (\underline{\pi}, \overline{\pi})$, both MMs and traders acquire information ($p > 0$ and $q > 0$).*
- *for $\pi \in (\overline{\pi}, 1)$ traders acquire information but MMs remain uninformed ($p = 0$ and $q > 0$). Moreover, as π tends to 1, the probability that traders acquire information tends to 0 as well.*

[9] The code used for calculating the equilibrium and simulating the prices in the following figures is available on the authors' websites.

[10] A similar result obtains if we instead fix x and vary y. However since, unlike x, y has a *direct* effect on traders' incentive to become informed, we feel that the comparative statics in terms of x is more meaningful.

We illustrate the proposition in Figure 2, panel B, for $c = 0.15$ and $y = 0$. The vertical axis shows information acquisition; the horizontal axis represents π. The logic behind Proposition 3 is as follows. MMs recoup the cost of information by trading with liquidity traders. The probability of informed market making thus increases with the share of traders hit by liquidity shocks. This effect in turn eliminates traders' incentives to become informed, since traders only recoup the cost of information by trading with uninformed MMs. Finally, in the limit as π tends to 1 all market orders proceed from speculators, pushing the ask price to 1 and the bid price to 0 and eliminating again traders' incentives to become informed in the first place. Information acquired by traders is thus non-monotonic in π.

5 Market Liquidity and Price Discovery

In this section we derive the implications of our analysis for market liquidity and price discovery. We set $y = 0$ for simplicity, such that $\pi = 1 - x$ and refer to π throughout in our results. Furthermore, we define c_1 to c_4 as in Figure 2: for $c < c_1$, $p > 0$ and $q = 0$; for $c \in (c_1, c_2)$, $p \in (0, 1)$ and $q \in (0, 1)$; for $c \in (c_2, c_3)$, $p \in (0, 1)$ and $q = 1$; for $c \in (c_3, c_4)$, $p = 0$ and $q = 1$.

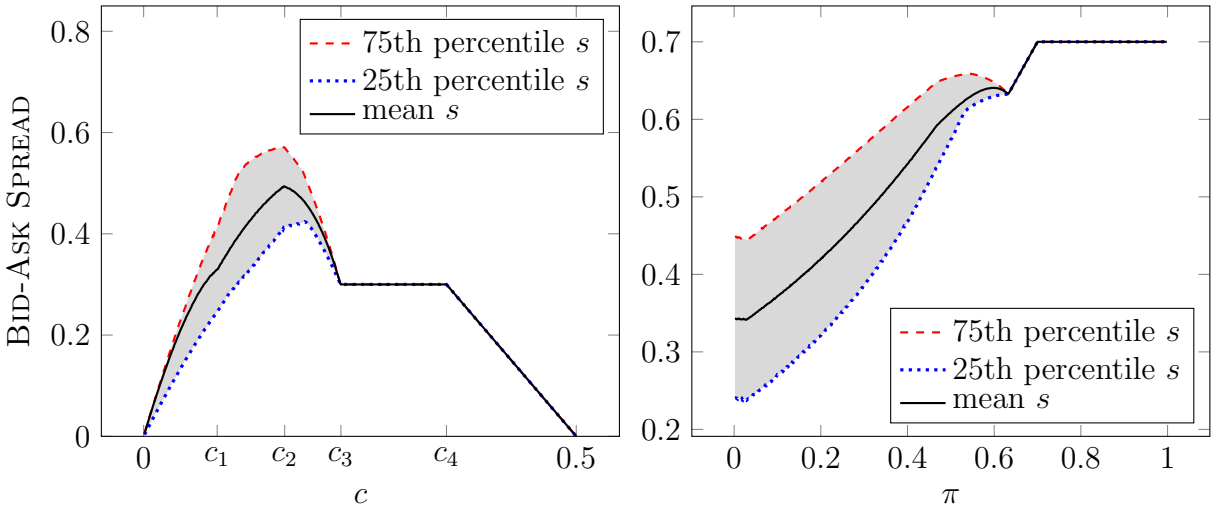
5.1 Market Liquidity

As usual in the literature, we measure market liquidity by the bid-ask spread, $s := \hat{a} - \hat{b}$. By Theorem 2, all agents remain uninformed for $c \geq \frac{1-y}{2}$. MMs thus face no adverse selection, and the bid-ask spread is 0. On the other hand, each MM becomes informed with probability converging to 1 as c tends to 0. Price competition between MMs thus drives the spreads to 0 in this case as well. These remarks yield the following corollary.

Corollary 2. *Market liquidity is non-monotonic in c and can be non-monotonic in π.*

Figure 4, panel A, illustrates the bid-ask spreads as a function of c for $\pi = 0.3$. The solid curve shows the expected spread. For c greater than c_4, MMs are uninformed, and the price-setting equilibrium is therefore in pure strategies, but q increases as c falls (see Figure 2, panel A). MMs therefore face increasing adverse selection, which pushes the spread upward. The flat portion corresponds to the Glosten-Milgrom region of information cost, $[c_3, c_4]$. In this range of cost, the MMs are uninformed, and q is constant at 1.

For c less than c_3, MMs acquire information with positive probability. From that point onwards, the MMs may be asymmetrically informed. The price-setting equilibrium is therefore



(A) Information Cost (B) Fraction of Speculators

Figure 4: Bid-Ask Spread

in mixed strategies. The distribution of the spread is given by the dashed curve, showing the 75th percentile spread, and by the dotted curve, indicating the 25th percentile spread. Notice that, moving from right to left, the expected spread goes on rising passed c_3. This follows from the fact that in the region of cost (c_2, c_3) speculators continue to be informed with probability 1, but now MMs acquire information with some probability as well. As a result, the adverse selection faced by uninformed MMs is unambiguously worse. In fact, the expected bid-ask spread is largest where the distance between the 75th and the 25th percentile spreads is maximum. These observations link our results to the empirical evidence showing that assets with more volatile prices exhibit higher spreads (Stoll, 1978; Chen *et al.*, 2007; Edwards *et al.*, 2007; Bao *et al.*, 2011).

Figure 4, panel B, shows the bid-ask spreads as a function of π for $c = 0.15$. The direct effect from increasing π is to worsen the adverse selection facing uninformed MMs, pushing the spreads upwards. However, in an intermediate region of π, increasing π lowers p, leaving q constant at 1 (see Figure 2, panel B). This region of π thus exhibits a countervailing effect on the spreads. In fact, as panel B of Figure 4 illustrates, the countervailing effect can be the dominant force: in that region, increasing π induces the expected spread to fall.

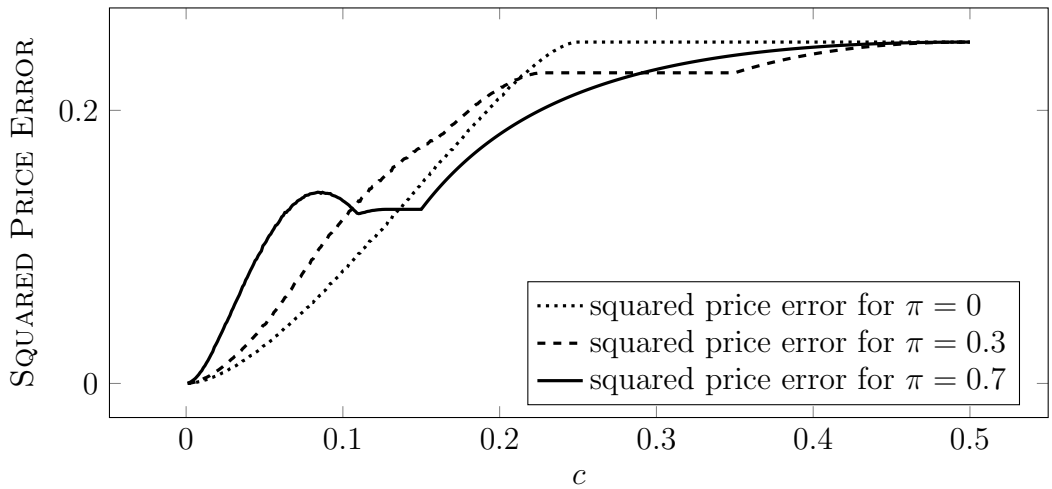


Figure 5: Price Discovery

5.2 Price Discovery

As usual in the literature price discovery (or inverse price discovery) is defined as the expected *squared price error*, $\mathbf{E}[(r - V)^2]$, where r is the realized price, i.e. $r = \hat{a}$ in case of a buy order, $r = \hat{b}$ in case of a sell order, and $r = \frac{\hat{a}+\hat{b}}{2}$ in case no trade occurs. Note that price discovery captures information contained in the quotes (if quotes contain any information at all), *and* information contained in the trade order (if the trade order contains any information at all). We will say that price discovery is quote driven (or dealer driven) if the trade order is uninformative, and that price discovery is order driven (or trader driven) if the quotes are uninformative. If speculators are uninformed, then all market orders proceed from liquidity traders. In this case, any information contained in the price r must come from the quotes reflecting the true asset value. If on the other hand MMs are uninformed then any information contained in r must proceed from informed speculators 'picking the right quote'. It is worth noting that order-driven price discovery is always bounded, due to the mass $1 - \pi$ of liquidity traders. By contrast quote-driven price discovery is unbounded: if all MMs were informed, competition would induce $\hat{a} = \hat{b} = V$ and, therefore, $r = V$ with probability 1. The next corollary summarizes the implications of Theorem 2 for price discovery.

Corollary 3. *Price discovery may be non-monotonic in c and π.*

Figure 5 shows price discovery (that is, $\mathbf{E}[(r - V)^2]$) with c on the horizontal axis. The dotted curve is for $\pi = 0$, the dashed curve for $\pi = 0.3$ and the solid curve for $\pi = 0.7$.

First, notice that the graph for $\pi = 0.7$ is non-monotonic in c. This illustrates the first

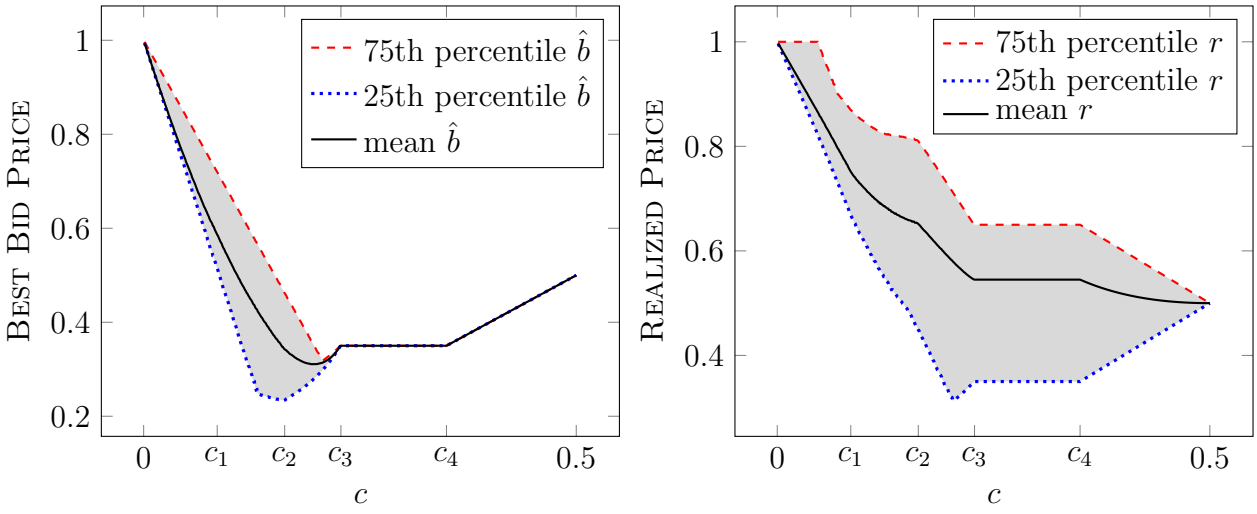
part of the corollary. The increase in the squared pricing error (as seen moving from right to left) starts exactly at $c = c_2$, i.e. at the point where traders start acquiring information with probability less than 1. Intuitively, the incentive for MMs to become informed is increasing in q (Theorem 1) and thus, as we move from right to left in the figure and q falls, p now increases at a slower rate (see Figure 2, panel A). The net effect is that price discovery worsens (pricing error is greater) as c falls.

Second, observe that, depending on the information cost, the squared pricing error may be increasing, decreasing or non-monotonic in π. For small information cost it is increasing, since increasing the fraction of liquidity traders enhances information acquired by MMs (without simultaneously reducing information acquired by traders; see Figure 2, panel B). The pattern is reversed for larger values of the information cost. However, in other areas, the squared pricing error is non-monotonic in π. For instance, for very high information cost only speculators acquire information, never MMs. In consequence, the squared pricing error is large for $\pi = 0$ since all trading is liquidity trading, improves for $\pi = 0.3$ as informed trading helps along price discovery, but deteriorates again for $\pi = 0.7$ since speculators eventually acquire less information for higher values of π (see Figure 2, panel B), thus reducing informed trading.

Finally, we illustrate with Figure 6 the rich interplay between market liquidity and price discovery. Panels A and B show, respectively, the realized price, r, and the best bid price, $\hat{b}$, as a function of c for $\pi = 0.3$ and $V = 1$. Starting from the right, at $c = c_3$ MMs start acquiring information, and this induces uninformed MMs to increase their spreads (in panel A, observe that $\hat{b}$ falls as c passes c_3). Price discovery from MMs therefore worsens immediately passed c_3. Yet panel B shows that the mean realized price moves closer to 1, that is, to the realization of V. The reason is that $q = 1$ immediately passed c_3. Hence, increasing the spread enhances price discovery on the part of speculators.

6 Conclusion

This paper analyzes information acquisition in dealer markets. We identify a one-sided strategic complementarity in information acquisition: the more informed traders are, the larger market makers' gain from becoming informed. When the cost of information is the same for all market participants, this complementarity uniquely pins down information acquisition. We show that for a range of information cost, information acquisition is as in the canonical model of Glosten and Milgrom (1985), that is, all traders are informed whereas none of the market



(a) Best Bid (b) Realized Price

Figure 6: Best Bid and Realized Price

makers are. However, other configurations arise as well, depending on the cost of information. In particular, for small cost, information acquisition is reversed. In that case, market makers are informed, but speculators are not. Similarly, when most traders are speculators, traders acquire information whereas MMs remain uninformed; by contrast, when most traders are liquidity traders, MMs acquire information and traders remain uninformed. We also explore market liquidity and price discovery. Most strikingly, increasing the cost of information or the fraction of liquidity traders can improve price discovery. Our results help explain several empirical regularities, such as dealer-driven price discovery, asymmetrically informed MMs, and the link between the size of spreads and price volatility.

Appendix A

Throughout the appendix, let $\Pi_n(b|\text{sell})$ (resp. $\overline{\Pi}_n(b|\text{sell})$) denote the expected profit of MMnU (resp. MMnH) bidding b, conditional on a sell order. Let also $\gamma := Pr(V = 0|\text{sell})$. Observe that in equilibrium $\gamma = (\frac{\pi q}{2} + \frac{1-\pi}{4})/(\frac{\pi q}{2} + \frac{1-\pi}{2})$.

Lemma 1. *Fix (p_1, p_2, q). There exists a unique price-setting equilibrium. If $p_1 = p_2 = 1$ then $a_n = b_n = V$, for all n. If $p_1 = p_2 = 0$ then $a_n = \frac{1}{2}\frac{1-\pi(1-2q)}{1-\pi(1-q)}$ and $b_n = \frac{1}{2}\frac{1-\pi}{1-\pi(1-q)}$, for all n. Otherwise the informed and the uninformed types of all MMs randomize, and there exist u and l_n, $n = 1, 2$, such that:*

1. *$\underline{\sigma}_n(b) = 1$ if $b \geq 0$ and $\underline{\sigma}_n(b) = 0$ if $b < 0$, for $n = 1, 2$;*
2. *$\inf\{supp\ \sigma_n \cup supp\ \overline{\sigma}_n\} = 0$ and $\sup\{supp\ \sigma_n \cup supp\ \overline{\sigma}_n\} = u$, for $n = 1, 2$;*
3. *$supp\ \sigma_n \cup supp\ \overline{\sigma}_n = [0, u]$, for $n = 1, 2$;*
4. *any atom in the strategies is at 0;*
5. *$u \in (0, 1)$;*
6. *$p_n \in (0, 1) \Rightarrow \sup\{supp\ \sigma_n\} = \inf\{supp\ \overline{\sigma}_n\} := l_n$, and $l_n < u$;*
7. *$1 > p_n > p_m > 0 \Rightarrow \frac{1}{2} > l_m > l_n > 0$.*

Proof: The proofs of existence and uniqueness are in Appendix B. The cases $p_1 = p_2 = 1$ and $p_1 = p_2 = 0$ are trivial, so we focus on the other cases, where $\max p_n > 0$ and $\min p_n < 1$. Here we show that in these cases, any price-setting equilibrium satisfies the properties listed in the lemma. Part 1 is straightforward, while Parts 2-4 follow from standard arguments Griesmer, Levitan and Shubik (1967); Engelbrecht-Wiggans *et al.* (1983). We show Parts 5-7.

Part 5: If $u = 0$ then one MM wins the bid with probability less than 1, say MMn. But then MMnH can bid ϵ and win with probability 1, thereby obtaining more profit. This shows that $u > 0$. We next show that $u < 1$. As $\min p_n < 1$ and $\max p_n > 0$, without loss of generality suppose $p_1 < 1$ while $p_2 > 0$. First, notice that $\sup\{\text{supp } \sigma_n\} \leq \mathbf{E}[V] = \frac{1}{2}$, for both MMs. But then $\overline{\Pi}_2(\frac{1}{2}|\text{sell}) \geq \frac{1}{2}(1 - p_1) > 0$. Hence MM2H bids so as to make strictly positive profit, which implies $\sup\{\text{supp } \overline{\sigma}_2\} < 1$. We have thus shown that $\sup\{\text{supp } \sigma_2 \cup \text{supp } \overline{\sigma}_2\} < 1$.

Part 6: If $\sup\{\text{supp } \sigma_n\} > \inf\{\text{supp } \overline{\sigma}_n\}$ then we can find $b'' > b'$ such that $b' \in \arg\max_b \overline{\Pi}_1(b|\text{sell})$ and $b'' \in \arg\max_b \Pi_1(b|\text{sell})$. Let $W_n(b|v) := Pr(\text{n wins}|b_n = b, V = v)$ and recall that $\gamma = Pr(V = 0|\text{sell})$. For all b:

$$\begin{aligned}\Pi_1(b|\text{sell}) &= \gamma W_1(b|0)(-b) + (1-\gamma)W_1(b|1)(1-b)\\ &= \gamma W_1(b|0)(-b) + (1-\gamma)\overline{\Pi}_1(b|\text{sell})\\ &\leq \gamma W_1(b|0)(-b) + (1-\gamma)\overline{\Pi}_1(b'|\text{sell}).\end{aligned}$$

But, by definition, $\Pi_1(b''|\text{sell}) \geq \Pi_1(b'|\text{sell})$. Hence $-b''W_1(b''|0) \geq -b'W_1(b'|0)$, from which $W_1(b''|0) < W_1(b'|0)$, since $b'' > b'$. This is a contradiction, since $W_1(\cdot|0)$ is increasing. Therefore, $\sup\{\text{supp } \sigma_n\} = \inf\{\text{supp } \overline{\sigma}_n\}$.

Part 7: By definition of u and as moreover there are no atoms at u, $\arg\max_b \overline{\Pi}_1(b|\text{sell}) = \arg\max_b \overline{\Pi}_2(b|\text{sell}) = 1-u$. Now observe that $l_n \geq l_m$ is impossible as otherwise by bidding l_m MMnH would win with strictly greater probability than MMmH with the same bid (and hence make strictly greater expected profit than MMmH); hence, $l_m > l_n$.

Next, $l_m = \sup\{\text{supp } \sigma_m\} \leq \mathbf{E}[V] = \frac{1}{2}$. Furthermore, combining Part 4, Part 6, and $p_m < 1$ implies $l_m < \frac{1}{2}$. All that is left to show is $l_n > 0$. Suppose instead that $l_n = 0$. Then σ_n has an atom at 0, in which case standard arguments yield $\Pi_m > \Pi_n = 0$. The proof of Proposition 1 below (case (b)) shows that this is impossible.

■

Proof of Proposition 1: Let $p_n > p_m$. By definition of u and as moreover there are no atoms at u, $\arg\max_b \overline{\Pi}_1(b|\text{sell}) = \arg\max_b \overline{\Pi}_2(b|\text{sell}) = 1-u$. Thus, $\overline{\Pi}_n = \overline{\Pi}_m$. That $\overline{\Pi}_n > \Pi_n$ is trivial. We are only left to show $\Pi_n > \Pi_m = 0$. We consider three cases separately: (a) $p_n = 1$, (b) p_n and p_m both in $(0,1)$, (c) $p_n < 1$ and $p_m = 0$.

Case (a): We have $\overline{\Pi}_n > 0$ and $0 \in \text{supp } \overline{\sigma}_n$. This requires MM$m$ to have an atom at 0. Standard arguments then yield $\Pi_m = 0$.

Case (b): Applying the arguments in the first part of the proof of Lemma 1.7 establishes

$l_n < l_m$. Thus $\sigma_n(l_m) = 1$ and

$$\begin{aligned}\Pi_m(l_m|\text{sell}) &= -\gamma(p_n + (1-p_n))l_m + (1-\gamma)(p_n\overline{\sigma}_n(l_m) + (1-p_n))(1-l_m)\\ &= -\gamma l_m + (1-\gamma)\overline{\Pi}_m(l_m|\text{sell}).\end{aligned}$$

γ is the probability that $V = 0$ conditional on a sell order; in this event, with a bid of l_m, MMm wins the order with probability 1; by contrast, conditional on $V = 1$, MMm wins the order with probability $\sigma_n(l_m) = 1$ if MMn is uninformed and with probability $\overline{\sigma}_n(l_m)$ if MMn is informed. At the same time,

$$\begin{aligned}\Pi_n(l_n|\text{sell}) &= -\gamma(p_m + (1-p_m)\sigma_m(l_n))l_n + (1-\gamma)(1-p_m)\sigma_m(l_n)(1-l_n)\\ &= -\gamma(p_m + (1-p_m)\sigma_m(l_n))l_n + (1-\gamma)\overline{\Pi}_n(l_n|\text{sell})\\ &> -\gamma l_m + (1-\gamma)\overline{\Pi}_n(l_n|\text{sell}).\end{aligned}$$

Since $\overline{\Pi}_n(l_n|\text{sell}) = \overline{\Pi}_m(l_m|\text{sell})$, we obtain $\Pi_n(l_n|\text{sell}) > \Pi_m(l_m|\text{sell})$. Therefore, $\Pi_n > \Pi_m$, since $l_m \in \text{supp}\ \sigma_m$ and $l_n \in \text{supp}\ \sigma_n$.

Lastly, $\Pi_m > 0$ implies that MMnU has an atom at 0, which in turn implies $\Pi_n = 0$ and so $\Pi_n < \Pi_m$, a contradiction. Hence, $\Pi_n > \Pi_m = 0$.

Case (c): The arguments for case (c) are analogous to those of case (b), where u is used instead of l_m.

■

Proof of Theorem 1:

Part 1: Let $p \in (0,1)$. Define $l := l_1 = l_2$. Then,

$$\Pi_n(l|\text{sell}) = -\gamma l + (1-\gamma)(1-p)(1-l). \tag{1}$$

With probability γ, $V = 0$, and $b_n = l$ wins the order with probability 1; with probability $1-\gamma$, $V = 1$, and $b_n = l$ wins the order if and only if MMm is uninformed (probability $1-p$). As $\Pi_n(l|\text{sell}) = 0$, we obtain

$$l = \frac{(1-\gamma)(1-p)}{\gamma + (1-\gamma)(1-p)}. \tag{2}$$

Next since, conditional on $V = 1$, $b_n = l$ wins the order if and only if MMm is uninformed, we have[11]

$$\overline{\Pi}_n(l|\text{sell}) = (1-p)(1-l).$$

As by symmetry of the bid and ask sides of the market $\overline{\Pi}_n = \frac{1-\pi}{2}\overline{\Pi}_n(l|\text{sell})$, using (2) to substitute for l yields

$$\overline{\Pi}_n(p,q) = \frac{1-\pi}{2}\left(\frac{\gamma(1-p)}{\gamma+(1-\gamma)(1-p)}\right). \tag{3}$$

Since γ is increasing in q, $\overline{\Pi}_n(p,q)$ is increasing in q and decreasing in p.

Part 2: Let $p \in [0,1]$. Define $F(b) := Pr(\hat{b} \leq b|V = 0)$. This is the distribution of the bid price facing the informed speculator who observed $V = 0$. Then $F(b) = 0$ if $b < 0$ and, if $b \geq 0$:

$$F(b) = (1-p)^2\sigma^2(b) + 2p(1-p)\sigma(b) + p^2.$$

Hence, for $b \in (0,l)$:

$$\frac{dF}{dp} = 2(1-p)^2\sigma\frac{d\sigma}{dp} + 2p(1-p)\frac{d\sigma}{dp} + 2[(1-\sigma)p + (1-p)(\sigma-\sigma^2)],$$

while

$$\frac{dF}{dq} = 2(1-p)^2\sigma\frac{d\sigma}{dq} + 2p(1-p)\frac{d\sigma}{dq}.$$

Observe that $\frac{d\sigma}{dp} > 0$ implies $\frac{dF}{dp} > 0$, and $\frac{d\sigma}{dq} > 0$ implies $\frac{dF}{dq} > 0$. We proceed to show that, for $b \in (0,l)$, $\frac{d\sigma}{dp} > 0$ and $\frac{d\sigma}{dq} > 0$. For $b \in [0,l]$:

$$\Pi_n(b|\text{sell}) = -\gamma[p + (1-p)\sigma(b)]b + (1-\gamma)(1-p)\sigma(b)(1-b) = 0.$$

This yields

$$\sigma(b) = \frac{\gamma p b}{(1-p)[(1-\gamma)(1-b) - \gamma b]}.$$

Therefore, $\frac{d\sigma}{dp} > 0$ and $\frac{d\sigma}{d\gamma} > 0$, for all $b \in (0,l)$. Since $\frac{d\gamma}{dq} > 0$, we have $\frac{d\sigma}{dp} > 0$ and $\frac{d\sigma}{dq} > 0$. Hence $\frac{dF}{dp} > 0$ and $\frac{dF}{dq} > 0$, for all $b \in (0,l)$. Since by symmetry of the bid and ask sides of the market, $\overline{\Pi}_S - \Pi_S = (1-y)\int b\, dF(b)$, part 2 of the theorem ensues.

■

[11] We again use the fact that if $p \in (0,1)$, then any atom in the bidding strategy is at 0.

Proof of Theorem 2:

Existence: Lemma 1 established that, given (p_1, p_2, q), price setting is uniquely determined in equilibrium. Proposition 1 showed that in equilibrium, $p_1 = p_2$. Define the set-valued function

$$\psi_i(p,q) := \begin{cases} 0 & \text{if } \ \overline{\Pi}_i(p,q) - c < \Pi_i(p,q) \\ [0,1] & \text{if } \ \overline{\Pi}_i(p,q) - c = \Pi_i(p,q) \\ 1 & \text{if } \overline{\Pi}_i(p,q) - c > \Pi_i(p,q). \end{cases}$$

For all $(p,q) \in [0,1] \times [0,1]$, $\psi_i(p,q)$ is convex. Next, let $(p^\dagger, q^\dagger) = \lim_{k\to\infty}(p_k, q_k)$. If $\psi_i(p^\dagger, q^\dagger) = [0,1]$ then trivially, for all $u_k \in \psi_i(p_k, q_k)$ and $u = \lim_{k\to\infty} u_k$, $u \in \psi_i(p^\dagger, q^\dagger)$. Furthermore, by Proposition 1, Π_i and $\overline{\Pi}_i$ are continuous. Therefore $\psi_i(p^\dagger, q^\dagger) = 0$ implies $\lim_{k\to\infty} \psi_i(p_k, q_k) = 0$, and $\psi_i(p^\dagger, q^\dagger) = 1$ implies $\lim_{k\to\infty} \psi_i(p_k, q_k) = 1$. Hence ψ_i has closed graph. We may therefore apply the Kakutani fixed point theorem to the correspondence $\psi_n \times \psi_S$. By construction, if $(p,q) \in \big(\psi_n(p,q), \psi_S(p,q)\big)$, an equilibrium exists in which MMs acquire information with probability p and the speculator acquires information with probability q.

Uniqueness: Let (p^*, q^*) denote an equilibrium information acquisition profile. Suppose $\overline{\Pi}_n(p^*, q^*) - \Pi_n(p^*, q^*) = \overline{\Pi}_S(p^*, q^*) - \Pi_S(p^*, q^*) = c$ (other cases are similar). Assume we can find other equilibrium values, (p', q'). If $p' = p^*$ then either $q' > q^*$ or $q' < q^*$. If $q' > q^*$ then $q' > 0$ and $\overline{\Pi}_S(p', q') - \Pi_S(p', q') < \overline{\Pi}_S(p^*, q^*) - \Pi_S(p^*, q^*) = c$, a contradiction. If $q' < q^*$, then $q' < 1$. Again we have a contradiction since $\overline{\Pi}_S(p', q') - \Pi_S(p', q') > \overline{\Pi}_S(p^*, q^*) - \Pi_S(p^*, q^*) = c$. Therefore, there is no other equilibrium with $p' = p^*$. Next, suppose $p' > p^*$. Then, $\overline{\Pi}_n(p', q^*) - \Pi_n(p', q^*) < c$ and $\overline{\Pi}_S(p', q^*) - \Pi_S(p', q^*) < c$. Since $p' > p^*$, we have $p' > 0$ and therefore, $\overline{\Pi}_n(p', q') - \Pi_n(p', q') \geq c$. This implies $q' > q^*$ and, therefore, $q' > 0$. This is a contradiction since in that case $\overline{\Pi}_S(p', q') - \Pi_n(p', q') < c$. Therefore, there is no equilibrium with $p' > p^*$. Finally, suppose $p' < p^*$. Then $\overline{\Pi}_n(p', q^*) - \Pi_n(p', q^*) > c$ and $\overline{\Pi}_S(p', q^*) - \Pi_S(p', q^*) > c$. Since $p' < p^*$, we have $p' < 1$. Hence $\overline{\Pi}_n(p', q') - \Pi_n(p', q') \leq c$, which in turn implies $q' < q^*$. But then $q' < 1$ whereas $\overline{\Pi}_S(p', q') - \Pi_S(p', q') > \overline{\Pi}_S(p^*, q^*) - \Pi_S(p^*, q^*) = c$, a contradiction. Therefore, there is no equilibrium with $p' < p^*$.

p^* is non-increasing in c: Follows from Theorem 1. Specifically, suppose $c_a < c_b$ and $p_a^* < p_b^*$.

Then $p_a^* < 1$, and so $\overline{\Pi}_n(p_a^*, q_a^*) - \Pi_n(p_a^*, q_a^*) \leq c_a < c_b$. This implies $q_a^* < q_b^*$. But we have a contradiction as in that case $\overline{\Pi}_S(p_a^*, q_a^*) - \Pi_S(p_a^*, q_a^*) > \overline{\Pi}_S(p_b^*, q_b^*) - \pi_S(p_b^*, q_b^*) \geq c_b > c_a$, which implies $q_a^* = 1$.

p^* tends to 1 as c tends to 0: As p^* is non-increasing in c, p^* converges as c tends to 0. Let z denote the limit. Then, using (3) yields

$$\lim_{c \to 0} \overline{\Pi}_n(p^*(c), q^*(c)) \geq \lim_{c \to 0} \overline{\Pi}_n(p^*(c), 0) = \frac{1-\pi}{2}\left(\frac{1-z}{2-z}\right).$$

Thus $z = 1$, or we have a contradiction.

$q^* = 0$ for c sufficiently small: Note that $p^* < 1$, for all $c > 0$. Furthermore, as p^* is non-increasing in c, we have $p^* \in (0, 1)$ for $c \in (0, \overline{c})$. In particular, in that interval of the cost, $\overline{\Pi}_n(p^*, q^*) = c$.

Using (3), setting $\overline{\Pi}_n(p, q) = c$ yields $p = p(q, c)$ where

$$p(q, c) := \frac{4c((1-q)\pi - 1) + (1-\pi)(1 - \pi(1-2q))}{(1-\pi)(1 - 2c - \pi(1-2q))}.$$

As $p(q, c)$ is increasing in q, we have $p(q, c) \geq p(0, c) = \frac{1-\pi-4c}{1-\pi-2c}$.

Next, observe that $l(p, q)$ derived in (2) is decreasing in p and in q. Hence, for $c \in (0, c_3)$, $l(p^*, q^*) \leq l(p(0, c), 0) = \frac{2c}{1-\pi}$.

Now, by symmetry of the bid and ask sides of the market,

$$\overline{\Pi}_S(p, q) - \Pi_S(p, q) = (1-y)\Big[2(1-p)p \int_0^l b \, d\sigma(b) + (1-p)^2 \int_0^l b \, d\sigma^2(b)\Big],$$

and, using part 2 of Theorem 1 (as well as the steps above), for $c \in (0, \overline{c})$:

$$\begin{aligned}
\overline{\Pi}_S(p^*, q^*) - \Pi_S(p^*, q^*) &\leq \overline{\Pi}_S(p(0, c), 0) - \Pi_S(p(0, c), 0) \\
&\leq (1-y)\Big[2\big(1 - p(0, c)\big)p(0, c)l(0, c) + \big(1 - p(0, c)\big)^2 l(0, c)\Big] \\
&= (1-y)\frac{8c^2(1 - \pi - 3c)}{(1-\pi)(1-\pi-2c)^2}.
\end{aligned}$$

Hence $\overline{\Pi}_S(p^*, q^*)\Pi_S(p^*, q^*) < c$ for c sufficiently small. This implies $q^* = 0$.

For c sufficiently large, traders acquire information but MMs remain uninformed: Applying Theorem 1 yields

$$\max_{(p,q)} \overline{\Pi}_S(p,q) - \Pi_S(p,q) = \overline{\Pi}_S(0,0) - \Pi_S(0,0) = (1-y)\frac{1}{2} > \frac{1}{4}(1-\pi) = \overline{\Pi}_n(0,0) - \Pi_n(0,0),$$

where the inequality follows from $y < \frac{2-x}{3-x}$.

■

Proof of Proposition 2: We have $\overline{\Pi}_S(0,1) - \Pi_S(0,1) = (1-y)\frac{1-\pi}{2}$, and $\overline{\Pi}_n(0,1) - \Pi_n(0,1) = (\frac{1+\pi}{2})(\frac{1-\pi}{2})$. Hence $\overline{\Pi}_S(0,1) - \Pi_S(0,1) > \overline{\Pi}_n(0,1) - \Pi_n(0,1)$ if and only if $y < \frac{x}{1+x}$.

■

Proof of Proposition 3: Let $y = 0$ and $x = 1 - \pi$ (other cases are similar). Then $\overline{c} = \frac{1-\pi^2}{4}$. It follows that $\frac{\partial \overline{c}}{\partial \pi} < 0$. Next, consider $\underline{c}$. This cost level is characterized by being the highest c such that $q = 0$. We make the following observations. (i) Notice that $\overline{\Pi}_S(p,0) - \Pi_S(p,0)$ does not depend on π. In particular, when $q = 0$ we have $\gamma = 1/2$ and prices do not depend on π. Therefore, traders' gain from acquiring information for $q = 0$ are independent of π. (ii) On the other hand, the MMs' gain from acquiring information at $q = 0$ is $\overline{\Pi}_n(p,0) - \Pi_n(p,0) = \frac{1-\pi}{2} \cdot \frac{1-p}{2-p}$, which is clearly decreasing in π. Let $\pi' > \pi$ and denote by $\overline{\Pi}_i$ and $\overline{\Pi}'_i$ the payoffs of agent i in the two cases, respectively. Suppose that $\underline{p}$ is the p that satisfies $\overline{\Pi}_n(\underline{p},0) - \Pi_n(\underline{p},0) = \overline{\Pi}_S(\underline{p},0) - \Pi_S(\underline{p},0) = \underline{c}$. It follows that $\overline{\Pi}'_n(\underline{p},0) - \Pi'_n(\underline{p},0) < \overline{\Pi}'_S(\underline{p},0) - \Pi'_S(\underline{p},0) = \underline{c}$. Thus, since both $\overline{\Pi}'_n - \Pi'_n$ and $\overline{\Pi}'_S - \Pi'_S$ are strictly decreasing in p, then for any $\underline{p}'$ such that $\overline{\Pi}'_n(\underline{p}',0) - \Pi'_n(\underline{p}',0) = \underline{c}$, we also have $\overline{\Pi}'_S(\underline{p}',0) - \Pi'_S(\underline{p}',0) > \underline{c}$. Hence, it is impossible that $q^* = 0$ for π' and $c = \underline{c}$. Since by Theorem 2 we have $q^* = 0$ for all $c < \underline{c}'$, where $\underline{c}'$ is the threshold that corresponds to π', it follows that $\underline{c}' < \underline{c}$.

■

Appendix B (for online publication)

Lemma 2. *A price-setting equilibrium exists for any q, p_1 and p_2.*

Proof: Let $\Pi_n(\text{sell})$ (resp. $\overline{\Pi}_n(\text{sell})$) denote the expected equilibrium profit of MMnU (resp. MMnH) conditional on a sell order, i.e. $\Pi_n(b|\text{sell})$ (resp. $\overline{\Pi}_n(b|\text{sell})$) evaluated at some b in the support of MMnU's (resp. MMnH's) strategy.

Suppose without loss of generality that $p_1 \geq p_2$. It is standard to show that we can find strategies that satisfy the properties of Lemma 1 and yield constant payoffs to the different types over the support of their strategies. Let f_n and g_n, respectively, denote one such set of strategies for MMnU and MMnH, respectively. We now show that no MM can profit from bidding outside the support of their proposed equilibrium strategy.

It is clear that no MM can ever strictly gain from bidding outside $[0, u]$. Now, consider MMnU, and denote the other MM by m. Bidding $b \in (l_n, u]$ yields payoff

$$\begin{aligned}
\Pi_n(b|\text{sell}) &= \gamma[p_m + (1-p_m)f_m(b)](0-b) + (1-\gamma)[p_m g_m(b) + (1-p_m)f_m(b)](1-b) && (4)\\
&= -\gamma[p_m + (1-p_m)f_m(b)]b + (1-\gamma)\overline{\Pi}_n(b|\text{sell}) && (5)\\
&\leq -\gamma[p_m + (1-p_m)f_m(l_n)]l_n + (1-\gamma)\overline{\Pi}_n(b|\text{sell}) && (6)\\
&= \Pi_n(l_n|\text{sell}) && (7)\\
&= \Pi_n(\text{sell}). && (8)
\end{aligned}$$

Here, (4) is by definition, (5) from substituting $\overline{\Pi}_n(b|\text{sell})$ into the expression, (6) since $b \in (l_n, u_n]$, (7) from the definition of $\Pi_n(b|\text{sell})$, and (8) since $\Pi_n(l_n|\text{sell}) = \Pi_n(\text{sell})$. Thus, an uninformed MM has no profitable deviation.

We now turn to MMnH. Notice that rewriting MMnU's payoffs, we have $\overline{\Pi}_n(b|\text{sell}) = \frac{\Pi_n(b|\text{sell})+\gamma[p_m+(1-p_m)f_m(b)]b}{1-\gamma}$. Thus, bidding $b \in [0, l_n)$ yields payoff

$$\begin{aligned}
\overline{\Pi}_n(b|\text{sell}) &= \frac{\Pi_n(b|\text{sell}) + \gamma[p_m + (1-p_m)f_m(b)]b}{1-\gamma} && (9)\\
&= \frac{\Pi_n(\text{sell}) + \gamma[p_m + (1-p_m)f_m(b)]b}{1-\gamma} && (10)\\
&< \frac{\Pi_n(\text{sell}) + \gamma[p_m + (1-p_m)f_m(l_n)]l_n}{1-\gamma} && (11)\\
&= \frac{\Pi_n(l_n|\text{sell}) + \gamma[p_m + (1-p_m)f_m(l_n)]l_n}{1-\gamma} && (12)\\
&= \overline{\Pi}_n(l_n|\text{sell}) && (13)\\
&= \overline{\Pi}_n(\text{sell}). && (14)
\end{aligned}$$

Here, (9) is by the result described immediately before the equations, (10) follows since b is in the support of MMnU, (11) since $b \in [0, l_n)$ and f_m is an increasing function, (12) since l_n is also in the support of MMnU, (13) from the above observation, and (14) since l_n is in the support of MMnH. Hence, the informed MM does not have a profitable deviation either.

We thus have an equilibrium. ■

Lemma 3. *The price-setting equilibrium is unique.*

Proof: Suppose without loss of generality that $p_1 \geq p_2$. Fix u. We now use Parts 1 to 7 of Lemma 1, which are shown above, to prove that the equilibrium is unique. Since there is no atom for $b > 0$, it must be that $\overline{\Pi}_n(\text{sell}) = 1 - u$. We know that the closure of the strategy supports of MMnU and MMnH overlap in exactly one point, which we denote l_n. We also know that $l_1 \leq l_2$. It follows that there exists a unique $\overline{\sigma}_2$ and l_2 which yield $\overline{\Pi}_1(b|\text{sell}) = 1-u$ for $b \in [l_2, u]$. To see this, notice that $l_1 \leq l_2$ implies that $\overline{\sigma}_2$ is defined entirely by the condition that $\overline{\Pi}_1(b|\text{sell}) = 1 - u$ for $b \in [l_2, u]$, and since there is no overlap in the support of σ_2 and $\overline{\sigma}_2$ (except at most at one b with zero probability mass) there is a unique $\overline{\sigma}_2$ which solves this. This in turn yields a unique l_2, since this is the highest b such that $\overline{\sigma}_2(b) = 0$. Thus, u uniquely defines a l_2, which we denote by l_2^u. Since $l_1 \leq l_2^u$, then $\overline{\Pi}_1(l_2^u|\text{sell}) = p_2(1 - l_2^u) = 1 - u$, and it follows that l_2^u is strictly increasing in u.

The MM who is informed with less probability must make zero profits when uninformed, which implies

$$\begin{aligned}\Pi_2(l_2^u|\text{sell}) &= -\gamma[p_1 + (1-p_1)\sigma_1(l_2^u)]l_2^u + (1-\gamma)[p_1\overline{\sigma}_1(l_2^u) + (1-p_1)\sigma_1(l_2^u)](1-l_2^u) \\ &= -\gamma l_2^u + (1-\gamma)(1-u) \\ &= 0.\end{aligned}$$

Since l_2^u is strictly increasing in u, and it is straightforward to see that $\Pi_2(l_2^1|\text{sell}) < 0$ and $\Pi_2(l_2^0|\text{sell}) > 0$, then there is exactly one u such that $\Pi_2(l_2^u|\text{sell}) = 0$.

Thus, the choice of u is unique given p_1 and p_2. It is then straightforward to show that $\Pi_1(\text{sell})$ is unique, and then given Parts 1 to 7 of Lemma 1, there exists a unique set of strategies that yield the required payoffs. In consequence, the equilibrium is unique.

■

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